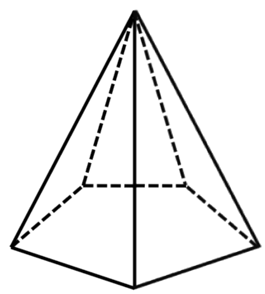
Exam Symmetry in Physics

Date	April 11, 2014
Room	5419.0013
Time	14:00 - 17:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the 3 exercises (18 in total) have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider a regular five-sided pyramid with a regular pentagon as base (see figure) and its symmetry group C_{5v}.



(a) Identify all transformations that leave this regular five-sided pyramid invariant.

(b) Show that C_{5v} is isomorphic to D_5, for instance using cycle notation.

(c) Argue, using geometrical arguments, that C_{5v} has four conjugacy classes.

(d) Determine the dimensions of all inequivalent irreps of C_{5v}.

(e) Construct explicitly the three-dimensional vector representation D^V for the two transformations that generate C_{5v} and extract a two-dimensional irrep from it.

(f) Construct the character table of C_{5v}. The irrep obtained in (e) may be used and it may be convenient to use $x \equiv \cos(2\pi/5) = -\frac{1}{4}(1-\sqrt{5})$ and $y \equiv \cos(4\pi/5) = -\frac{1}{4}(1+\sqrt{5})$, that satisfy $x^2 + y^2 = \frac{3}{4}$ and $xy = -\frac{1}{4}$.

(g) Decompose D^V of C_{5v} into irreps and use this to conclude whether this group allows in principle for an invariant three-dimensional vector, such as an electric dipole moment.

Exercise 2

Consider a non-isotropic medium with a conductivity tensor $\sigma_{ij} \neq \delta_{ij}$.

(a) Show using the transformation properties of the equation $j_i = \sigma_{ij} E_j$ that σ_{ij} transforms according to the direct product representation $D^V \otimes D^V$.

(b) Show that if σ_{ij} is (anti-)symmetric, then it remains (anti-)symmetric under orthogonal transformations.

(c) Show that σ_{ij} is invariant if it satisfies $\sigma D^V = D^V \sigma$.

(d) Assuming the medium has a C_3 symmetry, determine the most general invariant conductivity tensor.

(e) Explain why in the case of $SO(3)$, in other words for an isotropic medium, the only invariant tensor is proportional to δ_{ij}.

(f) Explain why the trace of a symmetric tensor σ_{ij} transforms as a scalar.

Exercise 3

Consider the group of rotations in three dimensions $SO(3)$.

(a) Specify the defining representation of $SO(3)$.

(b) Write down the elements of the subgroup of rotations around the x-axis in the defining representation.

(c) Write down the corresponding matrices in a spherical basis.

(d) Specify the three complex irreps of $SO(2)$ that appear in these matrices.

(e) Give an example of a physical system with an $SO(3)$ symmetry and give an example of a system that is not $SO(3)$ invariant itself but that has an $SO(3)$ invariant potential.